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A Bayesian Analysis of Record Statistics from the Rayleigh Model under Balanced Loss Functions

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Abstract

This paper develops a Bayesian estimation under balanced loss function for the parameter and some survival time parameters e.g. reliability and hazard functions of the Rayleigh distribution based on upper record values. non Linear programming is used to obtain the best values ω_1 and ω_2 of the balanced loss function of the form $L^q{}_{\rho,\omega,\delta_0}(\xi(\theta);\delta) = \omega_1 q(\theta)\rho(\delta_0,\delta) + \omega_2 q(\theta)\rho(\xi(\theta);\delta), \quad \omega_2 = 1-\omega_1,$. Our computations are based on the balanced loss function which contains the symmetric and asymmetric loss functions as special cases. Comparisons are made between Bayesian and maximum likelihood estimators via Monte Carlo simulation.

Keywords: Rayleigh distribution; Bayes estimates; Balanced loss function; Record Statistics; non Linear programming.

1. Introduction

The Rayleigh distribution is a special case of the Weibull distribution, which provides a population model useful in several areas of statistics including life testing and reliability which age with time as its failure rate is a linear function of time. Estimations predictions and some inferences concerning the Rayleigh distribution have been discussed by many authors, i.e., Howlader and Hossain[1], Mostert et al. [2] , Fernndez [3], Soliman [4] , and Lee, et.al. [5] The probability density function *(pdf)* and cumulative distribution function*(cdf)* of the Rayleigh distribution, respectively, are given by

$$f(x) = 2\alpha x \exp[-\alpha x^2], \qquad x \geq 0;\ \alpha > 0. \qquad (1)$$

and

$$f(x) = 1 - \exp[-\alpha x^2], \qquad x \geq 0;\ \alpha > 0. \qquad (2)$$

Also, The reliability function *R(t),* and the hazard (instantaneous failure rate) function *H(t)* at
mission time t for the Rayleigh distribution are given by

$$R(t) = \exp[-\alpha t^2], \qquad t \geq 0, \qquad (3)$$

and

$$H(t) = 2\alpha t, \qquad t \geq 0, \qquad (4)$$

Record values arise naturally in many real life applications involving data relating to sport, weather and life testing studies. Many authors have been studied record values and associated statistics, for example; see, Nagaraja [6], Ahsanullah [7], Arnold, et.al. [8],. Some inferential methods based on record values for the Rayleigh, Weibull, Inverse Weibull, Exponentiated Family, Lomax and Inverse Rayleigh, function distributions are studied by Balakrishnan and Chan [9],. Shojaee et al [10], Ahmed et al [11], Sultan [12], Sultan, et al [13], Asgharzadeh, and, Fallah.. [14], Shawky.and Badr [15] .

Let $X_1, X_2, X_3, \cdots\cdots X_n$ a sequence of independent and identically distributed (iid)random variables with *cdf F(x) and pdf f (x).* Set $Y_m = \max(X_1, X_2, X_3, \cdots\cdots X_m)$, $m \geq 1$,we say that X_j is an upper record and denoted by $X_{U(i)}$ if $Y_j > Y_{j-1}$, $j > 1$ Assuming that $X_{U(1)}, X_{U(2)}, X_{U(3)}, \cdots\cdots X_{U(m)}$ are the first m upper record values arising from a sequence $\{X_i\}$ of iid Rayleigh variables with *pdf* is given, by (1).

In this paper, non Linear programming is used to obtain the best values ω_1 and ω_2 of the balanced loss function we obtain and compare several techniques of estimation based on record statistics for the Rayleigh distribution. As well as the survival time parameters, namely the hazard and Reliability functions. Section 2 Record Values and the Maximum likelihood estimators of the parameter, the reliability and hazard functions. In Section 3, the Bayes estimators of the parameters, the reliability and hazard functions are derived. This is done using the gamma conjugate prior . The Bayes estimates are obtained using both the symmetric loss function (BSEL.) and the asymmetric loss function Balanced linear-exponential (BLINEX)).In section 4, a simulation study is discussed and results are presented. Concluding remarks are presented in section 5.

2. *Record Values and Maximum Likelihood Estimation (MLE)*

Suppose we observed the first m upper record values each of which has the Rayleigh distribution whose pdf and cdf are, respectively, given by (1) and (2). Based on those upper record values, for simplicity of notation, we will use x_i instead of $X_{U(i)}$ We have the joint density function of the first m upper record values $x \equiv x_{U(1)}, x_{U(2)}, x_{U(3)}, \cdots\cdots x_{U(m)}$ is given (see Arnold et al., 1998) by

$$f_{1,2,3,\ldots\ldots m}\left(X_{U(1)},X_{U(2)},X_{U(3)},\cdots\cdots X_{U(m)}\right)=f\left(x_{U(m)}\right)\prod_{i=1}^{m-1}\frac{f\left(x_{U(i)}\right)}{1-F\left(x_{U(i)}\right)},$$

$$0\le X_{U(1)}<X_{U(2)}<X_{U(3)},\cdots\cdots<X_{U(m)}<\infty, \quad \textbf{(5)}$$

Where $f(\cdot)$, *and* $F(\cdot)$ are given, respectively, by (1) and (2) after replacing x by $x_{U(i)}$ The likelihood function based on the m upper record values x is given by

$$\ell(\alpha/x)=(2\alpha)^m u\ \exp[-\alpha T_m], \qquad u=\prod_{i=1}^{m}x_{U(i)} \quad , \quad T_m=x^2{}_{U(} \quad \textbf{(6)}$$

and the log-likelihood function may be written as

$$L(\alpha/x)\equiv Ln(\ell)=m\,Ln(2\alpha)-\alpha T_m+\sum_{i=1}^{m}Ln\left(x_{U(i)}\right), \quad \textbf{(7)}$$

we obtain the estimators of by differentiating (7) with respect to the parameter α and equating to zero, then the maximum likelihood estimate (MLE), under upper record value, say $\hat{\alpha}_{ML}$, is given by

$$\hat{\alpha}_{ML}=\frac{m}{T_m} \quad \textbf{(8)}$$

where T_m is given by equation (6).

By the invariance property of the maximum likelihood estimator, we can obtain the maximum likelihood estimator of reliability function $\hat{R}(t)_{ML}$ and $\hat{H}(t)_{ML}$ of *R(t)* and *H(t)* are given by (3) and (4) after replacing by $\hat{\alpha}_{ML}$ in the form

$$\hat{R}_{ML}(t)=\exp\left[-\hat{\alpha}_{ML}t^2\right], \qquad t\ge 0, \quad \textbf{(9)}$$

$$\hat{H}_{ML}(t)=2\,\hat{\alpha}\,t_{ML} \qquad t\ge 0, \quad \textbf{(10)}$$

3. *Bayes Estimation*

In this section, we derive the Bayes estimates of the parameter (α), the reliability R(t) and the hazard functions *H(t)* of the Rayleigh distribution. We consider we use the balanced loss functions (BLF). It is assumed a $gamma(\delta,\beta)$ be a conjugate prior for (α) as

$$g(\alpha)=\frac{\beta^{\delta}}{\Gamma(\delta)}\alpha^{\delta-1}\exp[-\beta\alpha], \qquad \alpha>0,\quad \beta,\delta>0 \qquad \textbf{(11)}$$

Combining the likelihood function in(6) with the prior pdf of α in (11), we get the posterior of α as

$$\pi(\alpha/\underline{x})=\frac{L(\alpha;\underline{x})g(\alpha)}{\int_0^{\infty}L(\alpha;\underline{x})g(\alpha)d\alpha}=\frac{v^{n+\delta}}{\Gamma(m+\delta)}\alpha^{m+\delta-1}\exp[-\alpha v] \qquad \alpha>0 \qquad \textbf{(12)}$$

$$\underline{x}=x_{U(1)},x_{U(2)},x_{U(3)}\cdots\cdots x_{U(n)} \quad and \quad v=(\beta+T_m) \qquad \textbf{(13)}$$

3. 1. the balanced loss function(BLF)

The balanced loss function introduced by Zellner [16]. Jafari Jozani et al.[17] introduced an extended class of BLF of the form $L^{q}{}_{\rho,\omega,\delta_0}(\xi(\theta);\delta)=\omega_1 q(\theta)\rho(\delta_0,\delta)+\omega_2 q(\theta)\rho(\xi(\theta);\delta),\omega_2=1-\omega_1$, where $q(\cdot)$ is a suitable positive weight function and $\rho(\xi(\theta);\delta)$ is an arbitrary loss function when estimating $\xi(\theta)$ by δ. The parameter δ_0 is a chosen priori estimator of $\xi(\theta)$, obtained for instance from the criterion of maximum likelihood, least squares or unbiasedness among others. They give a general Bayesian connection between the case of $\omega_1>0$ and $\omega_1=0$ where $0\le\omega_1<1$. By choosing $\rho(\xi(\theta);\delta)=(\delta-\xi(\theta))^2$ and $q(\theta)=1$, the BLF reduced to the balanced squared error loss (BSEL) function, used by Ahmadi et al.(2009a,b), .[18] .[19] in the form $L_{\omega,\delta_0}(\xi(\theta);\delta)=\omega_1(\delta-\delta_0)^2+\omega_2\rho(\delta-\xi(\theta))^2$ and the corresponding Bayes estimate of the function $\xi(\theta)$ is given by

$$\delta_{\omega,\xi,\delta_0}(\underline{x})=\omega_1\delta_0+\omega_2 \mathrm{E}(\zeta(\theta)/\underline{x}) \qquad \textbf{(14)}$$

Also, by choosing $\rho(\xi(\theta),\delta)=\exp[c(\delta-\xi(\theta))]-c(\delta-\xi(\theta))-1$ and $q(\theta)=1$ we get the balanced (BLINEX) loss function written as

$$L_{C,\omega,\xi_0}(\underline{x})=\omega_1\left[Exp[c(\delta-\delta_0)]-c(\delta-\delta_0)-1\right]+\omega_2\left[Exp(c(\delta-\xi(\theta)))-c(\delta-\xi(\theta))-1\right]$$

for which the Bayes estimate of $\xi(\theta)$ takes the form

$$\delta_{c,\omega,\xi,\delta_0}(\underline{x})=-\frac{1}{c}Ln[\omega_1 Exp[-c\delta_0]+\omega_2\mathrm{E}(\exp[-c\xi(\theta)]/\underline{x})] \qquad \textbf{(15)}$$

where $c\neq 0$ is the shape parameter of BLINEX loss function.

3.1.1. Estimates Based on **balanced** *Squared Error Loss Function(BSEL)*

based on BSEL function, and by using (14) the Bayes estimate of a function $\lambda\equiv\alpha$, $R(t)$ *or* $H(t)$ is given by

$$\hat{\lambda}_{BSEL}=\omega_1\hat{\lambda}_{ML}+\omega_2\mathrm{E}((\lambda)/\underline{x}) \qquad \textbf{(16)}$$

where $\hat{\lambda}_{ML}$ is the ML estimate of $\lambda\equiv\alpha$, $R(t)$ *or* $H(t)$ and $\mathrm{E}((\lambda)/\underline{x})$ can be obtained using

$$\mathrm{E}(\lambda/\underline{x})=\int_0^\infty \lambda\pi(\alpha/\underline{x})\,d\alpha$$

Under BSEL loss function, and by using (16), the Bayes estimator $\hat{\alpha}_{BSEL}$ for α is

$$\hat{\alpha}_{BSEL}=\omega_1\hat{\alpha}_{ML}+\omega_2\mathrm{E}(\alpha/\underline{x}) \qquad \textbf{(17)}$$

where $\hat{\alpha}_{ML}$ is the ML estimate of α, which can be obtained using (8) and $\mathrm{E}(\alpha/\underline{x})$ can be obtained using

$$\mathrm{E}(\alpha/\underline{x}) = \int_0^\infty \alpha\pi(\alpha/\underline{x})$$

$$=\int_0^\infty \frac{1}{\Gamma(m+\delta)}v^{m+\delta}\alpha^{m+\delta}\exp[-\alpha v]\,d\alpha = \frac{(m+\delta)}{v} \qquad \textbf{(18)}$$

Also, the Bayes estimates $\hat{R}_{BSEL}$ and $\hat{H}_{BSEL}$ of the reliability *R(t)* and the HRF's *H(t)* at a mission time t relative to balanced Squared Error Loss Function are

$$\hat{R}_{BSEL} = \omega_1 \hat{R}(t)_{ML} + \omega_2 \mathrm{E}\left(\left(R(t)\right)/\underline{x}\right) \quad \textbf{(19)}$$

where $\hat{R}(t)_{ML}$ is the ML estimate of $R(t)$ which can be obtained using (9) and $\mathrm{E}\left(R(t)/\underline{x}\right)$ can be obtained using

$$\mathrm{E}\left(R(t)/\underline{x}\right) = \int_{\forall\alpha} \frac{1}{\Gamma(m+\delta)} \exp\left[-\alpha t^2\right] v^{m+\delta} \alpha^{m+\delta-1} \exp\left[-\alpha v\right] d\alpha$$

$$= \left(1 + \frac{t^2}{v}\right)^{-(m+\delta)} \quad \textbf{(20)}$$

and

$$\hat{H}_{BSEL} = \omega_1 \hat{H}(t)_{ML} + \omega_2 \mathrm{E}\left(\left(H(t)\right)/\underline{x}\right)$$

where $\hat{H}(t)_{ML}$ is the ML estimate of $H(t)$ $R(t)$ which can be obtained using (10) and $\mathrm{E}(\mid H \mid (t)/\underline{x})$ can be obtained using

$$\mathrm{E}\left(H(t)/\underline{x}\right) = \int_{\forall\alpha} 2\alpha t \; \pi\left(\alpha/\underline{x}\right) d\alpha$$

$$= 2t\left(\frac{m+\delta}{v}\right) \quad \textbf{(21)}$$

Where v is given in (13).

To find the best values ω_1 and ω_2 of the balanced loss function in the estimator $\hat{\lambda}_{BSEL} = \omega_1 \hat{\lambda}_{ML} + \omega_2 \mathrm{E}\left((\lambda)/\underline{x}\right)$ we need to solve the following optimization that minimizes its mean square error

$$MSE\left(\hat{\lambda}_{BSEL}\right) = \mathrm{E}\left(\hat{\lambda}_{BSEL} - \lambda\right)^2 = \mathrm{E}[(\omega_1 \hat{\lambda}_{ML} + \omega_2 \mathrm{E}\left((\lambda)/\underline{x}\right) - \lambda)]^2$$

that minimizes MSE Have been obtained by non ω_1 The value of linear programming

$$Minimize: MSE\left(\hat{\lambda}_{BSEL}\right) = E\left(\hat{\lambda}_{BSEL} - \lambda\right) = E[(\omega_1 \hat{\lambda}_{ML} + \omega_2 E((\lambda)/\underline{x}) - \lambda)]^2$$

subject to:

$$\omega_1 + \omega_2 = 1,$$
$$0 \le \omega_1 < 1,$$
$$0 \le \omega_2 < 1.$$

3.1.2. Estimates Based on balanced LINEX Error Loss Function(BLINEX)

based on the BLINEX loss function, the Bayes estimate the function λ is obtained by using (15) and written as

$$\hat{\lambda}_{BLINEX} = -\frac{1}{c} Ln\left[\omega_1 Exp\left[-c\hat{\lambda}_{ML}\right] + \omega_2 E\left(\exp[-c\lambda]/\underline{x}\right)\right] \qquad \textbf{(22)}$$

where $\hat{\lambda}_{ML}$ is the ML estimate of λ. Under the BLINEX loss function, and by using (22), the Bayes estimator $\hat{\alpha}_{BLINX}$ for α is given by

$$\hat{\alpha}_{Blinex} = -\frac{1}{c} Ln[\omega_1 Exp[-c\hat{\alpha}_{ML}] + \omega_2 E(\exp[-c\alpha]/\underline{x})] \qquad \textbf{(23)}$$

where $\hat{\alpha}_{ML}$ is the ML estimate of α
and $E(\exp[-c\,\alpha]/\underline{x})$ can be obtained using

$$E(\exp[-c\,\alpha]/\underline{x}) = \int_{\forall\alpha} \exp[-c\,\alpha]\pi(\alpha/\underline{x})d\alpha$$

$$= \int_0^\infty \frac{1}{\Gamma(m+\delta)} v^{m+\delta} \alpha^{m+\delta=1} \exp[-\alpha(v+c)]d\alpha \qquad (24)$$

$$= \left(1 + \frac{c}{v}\right)^{-(m+\delta)}$$

Also, the Bayes estimates $\hat{R}_{BSEL}$ and $\hat{H}_{BSEL}$ of the reliability *R(t)* and the HRF's *H(t)* at a mission time t relative to balanced LINEX Loss Function $\hat{R}_{BSEL} = \omega_1 \hat{R}(t)_{ML} + \omega_2 \mathrm{E}((-c\,R(t))/\underline{x})$

where $\hat{R}(t)_{ML}$ is the ML estimate of $R(t)$ and $\mathrm{E}(\exp[-c\,R(t)]/\underline{x})$ can be obtained using

$$\mathrm{E}(\exp[-c\,R(t)]/\underline{x}) = \int_{\forall\alpha} \exp[-c\,R(t)]\pi(\alpha/\underline{x})d\alpha$$

$$= \int_0^\infty \frac{1}{\Gamma(m+\delta)} v^{m+\delta}\alpha^{m+\delta-1}\exp[-c\exp[-\alpha t^2]\exp[-\alpha v]]d\alpha$$

$$= \sum_{i=0}^{\infty}\frac{(-c)^i}{i!}\left(1+\frac{i t^2}{v}\right)^{-(m+\delta)} \qquad (25)$$

and

$$\hat{H}_{BSEL} = \omega_1 \hat{H}(t)_{ML} + \omega_2 \mathrm{E}((-c\,H(t))/\underline{x})$$

$$\mathrm{E}(\exp[-c\,H(t)]/\underline{x}) = \int_{\forall\alpha} \exp[-c\,H(t)]\pi(\alpha/\underline{x})d\alpha$$

$$= \int_0^\infty \frac{1}{\Gamma(m+\delta)} v^{m+\delta}\alpha^{m+\delta-1}\exp[-\alpha(v+2tc)]d\alpha$$

$$= \left(1+\frac{2ct}{v}\right) \qquad (26)$$

Where v is given in (13).

Our first estimator is based on finding ω_1 that the minimizes the mean square errors Here we are looking ω_1 and ω_2 in the estimator

$$\hat{\lambda}_{BLNEX} = -\frac{1}{c}Ln[\omega_1 Exp[-c\lambda_{ML}] + \omega_2 \mathrm{E}(\exp[-c\lambda]/\underline{x})]$$

that minimizes its mean square error

$$MSE(\hat{\lambda}_{BLINX}) = (-\frac{1}{c}Ln[\omega_1 Exp[-c\lambda_{ML}] + \omega_2 \mathrm{E}(\exp[-c\lambda]/\underline{x})] - \lambda)^2$$

The value of ω_1 that minimizes MSE Have been obtained by non linear programming as following:

$$Minimize : MSE\left(\hat{\lambda}_{BSEL}\right)= E\left(\hat{\lambda}_{BSEL} - \lambda\right)= E[(\omega_1 \lambda_{ML} + \omega_2 E\left((\lambda)/\underline{x}\right) - \lambda)]^2$$

$$Minimize\, MSE\left(\hat{\lambda}_{BLINX}\right) = \left(-\frac{1}{c} Ln\left[\omega_1 Exp\left[-c\lambda_{ML}\right] + \omega_2 E\left(\exp\left[-c\lambda\right]/\underline{x}\right)\right] - \lambda\right)^2$$

subject to:

$$\omega_1 + \omega_2 = 1$$

$$0 \leq \omega_1 < 1$$

$$0 \leq \omega_2 < 1,$$

BY solving this non linear programming problem (using mathematica5.2,proc LP)

4. Simulation Study and Comparisons

We obtained, in the above Sections, Bayesian and non-Bayesian estimates for the shape parameter (α), reliability, $R(t)$, and failure rate, $H(t)$, functions of the Rayleigh distribution. We adopted the squared error loss, LINEX loss and balanced loss functions. The MLE's are also obtained. In order to compare the ML and Bayes estimates, we calculate the mean square error for each estimate according the following steps:

1. For given values (δ, β), we generate a random value α . from the prior pdf (11).
2. By using the value α from step 1, we generate m , (m= 4, 5, 6, 7) upper record values from Rayleigh distribution whose pdf is given by equation (1),
3. The different estimates of α , $R(t)$ and $H(t)$ at time t (chosen to be 0 .4) are computed.
4. Steps 1 to 3 are repeated 10,000 times and the mean square error (MSE) for each estimate (say $\hat{\theta}$) was calculated by using

$$MSE\left(\hat{\theta}\right)=\frac{1}{10000}\sum_{i=1}^{10000}\left(\hat{\theta}_i-\theta\right)^2$$

where $\hat{\theta}$ is the estimate at the i^{th} run.
Table 1,2 and 3 given below shows the mean squared error of the different estimates based on
10000 runs of Monte Carlo simulation and records up to 7.

Table 1. MSEs of the estimates of α

(δ,β) α	m	ML	BSEL		BLINEX					
			$\hat{\alpha}_{SEL}$	$\hat{\alpha}_{BSEL}$	$\hat{\alpha}_{LINEX}$			$\hat{\alpha}_{BLINEX}$		
			$\omega_1 = 0$	Best ω_1 NLP	$\omega_1 = 0$			Best ω_1 NLP		
					C=0.0001	$C=1$	$C=2$	C=0.0001	$C=1$	$C=2$
(2,1) 1.383	4	1.9001	0.5710	0.5574	0.5710	0.2523	0.1566	0.5574	0.2523	0.1512
	5	1.0795	0.4778	0.4656	0.4778	0.2371	0.1458	0.4656	0.2371	0.1405
	6	0.8066	0.4095	0.3980	0.4095	0.2103	0.1304	0.3980	0.2103	0.1245
	7	0.5427	0.3463	0.3330	0.3463	0.1904	0.1137	0.3330	0.1904	0.1086
(2,2) 3.004	4	7.9592	1.4099	0.3716	1.4099	1.9849	2.4671	0.3716	0.3717	0.3631
	5	5.3755	1.1503	0.2818	1.1503	1.6530	2.0962	0.2818	0.2817	0.2673
	6	3.3381	0.9168	0.1723	0.9168	1.3595	1.7680	0.1723	0.1718	0.1615
	7	2.5492	0.6932	0.0860	0.6932	1.0804	1.4565	0.0860	0.0837	0.0830

Table 2. MSEs of the estimates of $R(t)$

(δ,β) α	m	ML	BSEL		BLINEX					
			$\hat{R}(t)_{SEL}$	$\hat{R}(t)_{BSEL}$	$\hat{R}(t)_{LINEX}$			$\hat{\alpha}_{BLINEX}$		
					$\omega_1 = 0$			Best ω_1 NLP		
			$\omega_1 = 0$	Best ω_1 NLP	$C=-1$	$C=1$	$C=2$	$C=-1$	$C=-1$	$C=-1$
(2,1) 1.383	4	0.0163	0.0066	0.0065	0.0062	0.0070	0.0080	0.0061	0.0068	0.0077
	5	0.0113	0.0057	0.0056	0.0053	0.0061	0.0066	0.0053	0.0059	0.0064
	6	0.0086	0.0049	0.0048	0.0046	0.0052	0.0054	0.0046	0.0050	0.0052
	7	0.0065	0.0043	0.0041	0.0040	0.0046	0.0049	0.0039	0.0044	0.0046
(2,2) 3.004	4	0.0287	0.0184	0.0045	0.0193	0.0175	0.0164	0.0045	0.0045	0.0043
	5	0.0223	0.0148	0.0033	0.0154	0.0141	0.0132	0.0032	0.0033	0.0031
	6	0.0165	0.0116	0.0020	0.0123	0.0110	0.0103	0.0019	0.0020	0.0018
	7	0.0135	0.0087	0.0009	0.0093	0.0082	0.0076	0.0009	0.0009	0.0009

Table 3. MSEs of the estimates of $H(t)$

(δ,β) α	m	ML	BSEL: $\hat{H}(t)_{SEL}$ $\omega_1=0$	BSEL: $\hat{H}(T)_{BSEL}$ Best ω_1 NLP	BLINEX: $\hat{H}(t)_{LINEX}$ $\omega_1=0$, $C=-1$	$\hat{H}(t)_{LINEX}$ $\omega_1=0$, $C=1$	$\hat{H}(t)_{LINEX}$ $\omega_1=0$, $C=2$	$\hat{H}(t)_{BLINEX}$ Best ω_1 NLP, $C=-1$	$\hat{H}(t)_{BLINEX}$ Best ω_1 NLP, $C=1$	$\hat{H}(t)_{BLINEX}$ Best ω_1 NLP, $C=2$
(2,1) 1.383	4	1.2165	0.3655	0.3568	0.8539	0.1855	0.1189	0.7214	0.1855	0.1178
	5	0.6909	0.3058	0.2980	0.6201	0.1717	0.1104	0.5133	0.1717	0.1093
	6	0.5162	0.2621	0.2547	0.4904	0.1514	0.0979	0.3995	0.1514	0.0967
	7	0.3473	0.2217	0.2131	0.3887	0.1370	0.1474	0.3048	0.1369	0.0863
(2,2) 3.004	4	5.0939	0.9024	0.2378	0.5904	1.2008	1.4583	0.2401	0.2379	0.2324
	5	3.4403	0.7362	0.1803	0.4749	0.9956	1.2297	0.1795	0.1803	0.1711
	6	2.1364	0.5868	0.1103	0.3794	0.8141	1.0278	0.1180	0.1100	0.1034
	7	1.6315	0.4436	0.0550	0.2808	0.6414	0.8362	0.0729	0.0536	0.0531

5. Concluding Remarks

In this paper we have presented. non Linear programming is used to obtain the best values ω_1 and ω_2 of the balanced loss function the Bayesian and non-Bayesian estimates of the parameter α , reliability, *R*(*t*), and failure rate, *H*(*t*), functions of the lifetimes follow the Rayleigh distribution. The estimation are conducted on the basis of upper record values. Bayes estimators, under balanced squared error loss, balanced LINEX loss functions. are derived. The MLE's are also obtained.

Our observations about the results are stated in the following points:

1. from all Tables shows that the Bayes estimates under balanced LINEX loss function have the smallest estimated MSE's as compared with the estimates under LINEX loss function, balanced Squared Error Loss Function, Squared Error Loss Function or MLE's. On the other hand the Bayes estimates under balanced Squared Error Loss Function have the smallest estimated MSE's as compared with the estimates under Squared Error Loss Function or MLE's. Also, the Bayes estimates under the LINEX loss function have the smallest estimated MSE's as compared with the estimates under Squared Error Loss Function. The results also show that the MSEs decreases as m increasing.
2. For estimating α in the case of small record sample size, it is recommended that one uses the Bayes method of estimation. Note that (see Tables 1) when the sample size is small ($m = 4$), the ML method has a large MSE.
3. To access the effect of the shape parameter of the asymmetric loss Function C we examine different values of C we see that if C is near to 0 then the Bayes estimates are almost the same as the estimates under SEL, (see Tables 1) This is one of the useful properties of working with the asymmetric loss functions.

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الملخص :

في هذا البحث تم تطوير أسلوب بيز تحت دالة الخسارة المتوازانه لتقدير معالم ودالة المعولية ودالة المخاطرة لتوزية ريلي بالاعتماد علي البيانات المسجلة العليا . حيث تم استخدام البرمجة الي الخطية لإيجاد أفضل قيمة لـ ω_1 و ω_2 لدالة الخسارة المتوازنة ذات الصيغة

$$L^{q}{}_{\rho,\omega,\delta_0}\left(\xi(\theta);\delta\right)=\omega_1 q(\theta)\rho(\delta_0,\delta)+\omega_2 q(\theta)\rho\left(\xi(\theta);\delta\right),\qquad \omega_2=1-\omega_1,$$

وتمت مقارنة مقدرات بيز مع مقدرات الإمكان الأعظم باستخدام دالة الخسارة المتوازنه ودوال الخسارة المتماثلة وغير المتماثلة باستخدام المحاكاة.